NUMERICAL INVESTIGATION OF THE THREE-DIMENSIONAL RADIATION FIELD IN HYPERSONIC FLOW OVER SEGMENTED BODIES

A. A. Kostuzik and A. N. Rumynskii

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The authors have made a numerical study of the three-dimensional radiation field in the shock layer on segmented bodies washed by a hypersonic inviscid, selectively radiating and absorbing gas.

Radiative heating of bodies in hypersonic flow has been the subject of many papers, but in an overwhelming majority of these an appreciable simplification is used in considering radiative heat transfer: the high-temperature shock layer on the windward body surfaces is assumed to be plane-parallel and locally one-dimensional; and the radiation field is also considered to be locally one-dimensional, which considerably facilitates numerical computations. By using a planar layer model one can obtain a quantitatively correct estimate of the radiative heating of the surface of smooth blunt bodies, but the accuracy of the estimate is unknown, as a rule, and, as was shown, for exmaple, in [1], there may be considerable error for spectral ranges for which the shock layer is transparent to radiation. The source of error is the difference in the shape of the radiating volume for a planar layer, and also the spatial nonuniformity of the distribution of the thermodynamic parameters of the gas. As far as the authors know, the existing literature contains no sufficiently complete analysis of the influence of these factors on the radiative heat transfer in the shock layer.

The present paper describes a numerical study of the three-dimensional radiative field in the shock layer in spatial hypersonic flow over bodies of segmented shape of an inviscid, non-heat-conducting, selectively radiating and absorbing gas. In calculating the radiant energy transfer we allow for the curvature of the shock layer surface and coupling of the radiative and the flow fields. The calculations are made for air, whose thermodynamic properties are taken from [2]. The flow in the shock layer is assumed to be locally in equilibrium, and we neglect precursor radiation and radiation from the body surface. The validity and the limits of application of these assumptions are discussed in detail in [3].

The spectral details of the radiant energy transfer are calculated on the 10-step absorption coefficient model of [4], based on the data of [5]. This model includes emission and absorption in the continuum, in weak atomic lines (evaluated by integration), and also in the wings of strong atomic lines. As was shown in [4], the calculated radiative heat flux in a hypersonic shock layer using this model does not differ by more than 10% from the value obtained by using a detailed description of the optical properties of the gas [5].

We consider radiant energy transfer in the shock layer in the absence of scattering. The spectral intensity of the radiative heat flux $I_\nu$ is determined by the radiative transfer equation, whose solution, allowing for the assumptions made, can be written in the form

$$I_\nu(S,\ \mathbf{l}) = B_\nu(S) - B_\nu(S^*)\exp(-\tau_\nu^*) - \int\limits_0^{\tau_\nu^*} \frac{dB_\nu(\tau_\nu)}{d\tau_\nu}\, d\exp(-\tau_\nu), \tag{1}$$

where $\tau_\nu(S,\ \mathbf{l}) = \int\limits_{s_0}^{s} k_\nu(s')\,ds'$; $\tau_\nu^* = \tau_\nu(S^*,\ \mathbf{l})$.

In terms of $I_\nu$ we can determine: the rate of influx of radiative thermal energy $q_R$ and the components of the radiative heat flux vector $\mathbf{H}$:

$$q_R(S) = \int\limits_0^\infty \varkappa_\nu(S)\Big[\int\limits_{(4\pi)} I_\nu(S,\ \mathbf{l})\,d\omega - 4\pi B_\nu(S)\Big]d\nu, \tag{2}$$

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$$H_i(S) = \int_0^\infty \int_{(4\pi)} I_\nu(S, \mathbf{l}) n_i d\omega d\nu, \quad i = 1, 2, 3. \tag{3}$$

To compute $q_R$ and **H**, Eqs. (1)-(3) are integrated numerically over the spatial variables and the frequency [6]. This approach to solving the problem meets with great computational effort, but with it one can evaluate radiative energy transfer in nonuniform gaseous volumes, such as the hypersonic shock layer, and study the effects of nonuniformity of the radiative field.

Let us consider the method as applied to numerical computation of the functions $I_\nu$, $q_R$, and **H**. We introduce the radiant flux intensity in the direction $\mathbf{l}_k$ for the m-th spectral interval:

$$I_{m,k}(S) = \int_{\nu_m}^{\nu_m+\Delta\nu_m} I_\nu(S, \mathbf{l}_k) d\nu,$$

where m = 1, 2, ..., M (here M = 10) is the spectral interval index, and k = 1, 2, ..., K is the spatial direction index. The quantity $I_{m,k}$ is determined from Eq. (1), of which the finite-difference approximation has the form

$$I_{m,k}(S) = B_{m,0} - B_{m,N} \cdot \exp(-\tau_{m,N}) - \sum_{n=1}^{N} B'_{m,n} [\exp(-\tau_{m,n}) - \exp(-\tau_{m,n-1})], \tag{4}$$

where

$$B_{m,n} = \int_{\nu_m}^{\nu_m+\Delta\nu_m} B_\nu(s_n) d\nu; \quad B'_{m,n} = \int_{\nu_m}^{\nu_m+\Delta\nu_m} (dB_\nu(\tau_\nu)/d\tau_\nu)|_{s=s_n} d\nu;$$

$$\tau_{m,n} = \int_{\nu_m}^{\nu_m+\Delta\nu_m} \tau_\nu(s_n, \mathbf{l}_k) d\nu;$$

n = 0, 1, ..., N is the index of a point on the ray $\mathbf{l}_k$, and the values n = 0 and n = N correspond to the points S and S*.

In carrying out the numerical computations we assume that the functions $\varkappa_\nu$ and $dB_\nu/d\tau_\nu$ are constant within the step $\Delta s_n$ of the integration along the ray $\mathbf{l}_k$.

To determine the functions $q_R$ and **H**, the space is divided into K = 400 equal solid angles within which $I_{m,k}$ is independent of direction and is determined from Eq. (4). The finite-difference equations for $q_R$ and components of the vector H are written in the following form:

$$q_R(S) = \sum_{m=1}^{M} \varkappa_m(S) \Big[\sum_{k=1}^{K} I_{m,k}(S) \Delta\omega - 4\pi B_m(S)\Big], \tag{5}$$

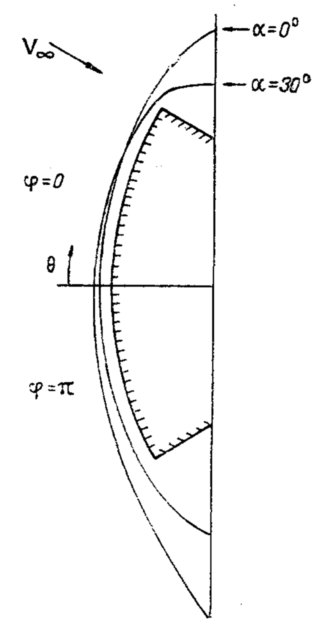


Fig. 1. Shape of the shock layer ($\alpha$ = 0 and 30°).

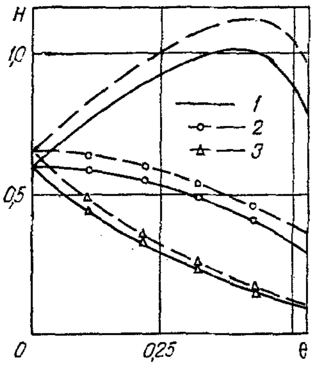


Fig. 2

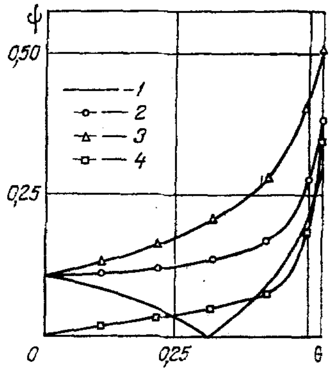


Fig. 3

Fig. 2. Distribution of radiative heat flux $H_r$ and $H_\delta$ (broken lines) over the surface of a segmented body in the planes $\varphi = 0$ (1), $\varphi = \pi/2$ (2), and $\varphi = \pi$ (3). The quantities $H_r$ and $H_\delta$ are referred to $H_{r\,max}$ ($\alpha$ = 30°).

Fig. 3. Distribution of the function $\psi$ over the surface of a segmented body in the planes $\varphi = 0$ (1), $\varphi = \pi/2$ (2), and $\varphi = \pi$ (3); 4) $\alpha = 0$.

$$H_i(S) = \sum_{m=1}^{M} \sum_{k=1}^{K} I_{m,k}(S)\, n_{i,k} \Delta\omega, \quad i = 1,\ 2,\ 3. \tag{6}$$

Here $\Delta\omega = 4\pi/K$, $\varkappa_m$ is the value of $\varkappa_\nu$ in the m-th spectral interval.

Equations (5) and (6) were obtained from Eqs. (2) and (3) by replacing integration over frequency and solid angle by summation over a finite number of intervals.

A time-dependent method is used to calculate the shock layer shape and the distribution of the gasdynamic functions in it. The following system of unsteady equations of radiative gasdynamics is solved:

$$\begin{gathered} \frac{d\mathbf{V}}{dt} + \frac{1}{\rho} \operatorname{grad} p = 0, \\ \frac{dp}{dt} + \rho a^2 \operatorname{div} \mathbf{V} + a^2 \left( \frac{\partial \rho}{\partial h} \right)_p q_R = 0, \\ \frac{dh}{dt} - \frac{1}{\rho} \frac{dp}{dt} - q_R = 0. \end{gathered} \tag{7}$$

To these equations we add the relations for the gas density $\rho = \rho(p, h)$ and the speed of sound $a = a(p, h)$. The rate of influx of radiant thermal energy $q_R$ is determined from Eq. (2).

We seek a solution in the region bounded by the body surface, the shock wave, and a certain surface located in the supersonic part of the shock layer and closing the computational region downstream. As boundary conditions we use: on the body surface the impermeability condition, and at the shock wave the Rankine–Hugoniot relations.

The steady-state flow field is determined by solving the system of equations (7) as $t \to \infty$. In the computations we used the unsteady mesh-characteristics method of Magomedov–Kholodov [7], as developed for the case of radiating gas flow in [8]. The method of obtaining the finite-difference equations approximating to the system (7) has been described in detail in [9] and is not discussed here.

We now consider some of the most interesting results of the numerical computations performed with the above technique.

We introduce the spherical coordinate system ($\theta$, r, $\varphi$), in which the angle $\theta$ is reckoned from the body axis of symmetry, the stagnation point lies in the plane $\varphi = 0$, and the coordinate origin coincides with the center of curvature of the body surface.

Figure 1 shows the shock layer shape for a segmented body with a semivertex angle of 30°, washed at an angle of attack of $\alpha = 0$ and 30°. Here and below the results shown were obtained for the following flow conditions: $V_\infty = 15$ km/sec, $\rho_\infty = 3.3 \cdot 10^{-4}$ kg/m³, $R_0 = 1$ m. As can be seen from Fig. 1, the shock standoff distance is small compared with $R_0$ over the entire frontal body surface, but the shock layer shape differs markedly from a plane layer.

A comparison of the results of the calculated radiative heat flux to the surface of the segment ($H_r$) with results calculated according to the planar layer model ($H_\delta$) shows that the difference between $H_\delta$ and $H_r$ is largest (23-26%) near the edge of the body, and that it does not exceed 10% on the greater part of the front surface. This can be seen from Fig. 2, which shows distributions of $H_r$ and $H_\delta$ along generators of the segment in the plane $\varphi = 0$, $\varphi = \pi/2$, and $\varphi = \pi$ for $\alpha = 30°$. The displacement of the point of maximum $H_r$ relative to the stagnation point is due to the sharp decrease of the shock standoff distance near the knee in the body surface (see Fig. 1).

The influence of the nonuniformity of the radiation field on the radiant energy transfer is shown in Fig. 3, which shows the distribution of the function $\psi$ corresponding to Fig. 2 — the angle between the radiative heat flux vector and the normal to the segment surface ($\alpha = 0$ and 30°). It can be seen that changing the angle of attack from 0 to 30° leads to an appreciable increase of the role of the tangential component of the **H** vector in the radiative energy transfer, but the radiative energy flux through the shock layer remains the dominant factor over the greater part of the front surface of the body, as it does in the case $\alpha = 0°$.

## NOTATION

$\nu$, frequency of the radiation; $I_\nu$, spectral intensity of the radiative heat flux; $B_\nu$, Planck function; $\tau_\nu$, optical thickness of the gas; $q_R$, rate of influx of the radiative heat energy; **H**, radiative heat flux vector; $H_i$, components of the vector **H** in an arbitrary orthogonal coordinate system (i = 1, 2, 3); $\varkappa_\nu$, $k_\nu$, mass and linear absorption coefficients, respectively, allowing for forced emission; $l$, unit vector giving the direction of radiation; S, space point; S*, point of intersection of the ray $l$ with the boundary of the radiating volume; $\omega$, solid angle; $n_i$, direction cosines of the vector $l$ (i = 1, 2, 3); s, coordinate along the ray $l$; k, space direction index (k = 1, 2, ..., K); m, spectral interval index (m = 1, 2, ..., M); n, index of the point of the ray (n = 0, 1, ..., N); $\Delta s_n$, mesh step on ray $l$; $\Delta\omega$, integration step in solid angle; t, **V**, $a$, $\rho$, p, and h, respectively, time, vector velocity, speed of sound, density, pressure, and enthalpy; the subscript $\infty$ denotes values of the parameters of the incident stream; $R_0$, nose blunting radius; $\alpha$, angle of attack; $\theta$, r, $\varphi$, spherical coordinate system; $H_r$, radiative heat flux to the body surface; $H_\delta$, radiative heat flux to the body surface, obtained from the planar layer model; $\psi$, angle between the vector **H** and the normal to the body surface.

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